# Gridlock Resolution in Payment Systems

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## INTRODUCTION

During the last few decades, most industrialised countries have introduced real-time gross settlement (RTGS) systems for settlement of large, time-critical inter-bank payments. Denmark was among the first; the DN Inquiry and Transfer System was introduced in 1981, and in 1999 DEBES, as part of TARGET[1], was launched for euro[2] payments. The two Danish systems were integrated in connection with the implementation of KRONOS on 19 November 2001.

In an RTGS system, payments are settled individually in real time, and the payments are final and irrevocable upon settlement. The advantage of RTGS systems is that they eliminate the credit risks and also the consequential potential systemic risk associated with other types of payment systems such as netting systems. A drawback of RTGS systems is that, compared to netting, the liquidity requirements increase when the payments are settled individually in real time. Mobilising the required liquidity imposes costs on the banks, and all other things being equal, the banks have an incentive to economise on liquidity and may prefer to wait for incoming payments before sending their own payments. This can lead to delays and *gridlocks,* i.e. situations where several payments each await settlement of the others.

This article describes the phenomenon of gridlock in relation to payment systems, and discusses a resolution mechanism that has been implemented in connection with KRONOS. The results of a number of simulations using actual data from Denmark and Finland are also presented in order to quantify the effects of the gridlock resolution mechanism.

## GRIDLOCKS IN PAYMENT SYSTEMS

From time to time, banks have insufficient liquidity on their settlement accounts at Danmarks Nationalbank to settle payments in KRONOS at the

[1] Trans-European Automated Real-Time Gross Settlement Express Transfer System.
[2] See Angelius, Hansen and Mærsk (1998), and Berg and Christensen (1999).

same pace as payment orders are received from customers or as a result of the banks' proprietary operations in the currency, securities and money markets. Insufficient liquidity to settle these transactions on an individual basis leads to settlement queues. Delays in payments might not only be costly for the bank with insufficient liquidity, but also to other banks because of the recycling of liquidity in an RTGS system. In most RTGS systems the majority of the liquidity used for settling payments comes in the form of incoming payments, and a delay in receiving these might cause liquidity problems for other banks in the system.

Such formations of queues are referred to as *gridlocks* if the formation of queues can be attributed to the requirement for payments to be settled individually. If the formation of queues can be attributed to a lack of liquidity, they are referred to as *deadlocks*. These concepts and a description of how gridlocks may be resolved are illustrated in the following example. General definitions of gridlock, deadlock and the gridlock resolution problem are given in the appendix.

## EXAMPLES OF GRIDLOCK AND DEADLOCK

Assume that there are three banks: A, B and C, which are to send payments to each other. Chart 1 illustrates three situations: no queue, gridlock and deadlock.

In the first instance, bank A has kr. 15 on its settlement account and must send exactly kr. 15 to bank B, which in turn has kr. 5 at its disposal and must send kr. 20 to bank C. Finally, bank C has kr. 10 on its settlement account and must send kr. 25 to bank A. In this situation the payment "circle" presents no problems, provided that bank A decides to send its payment.

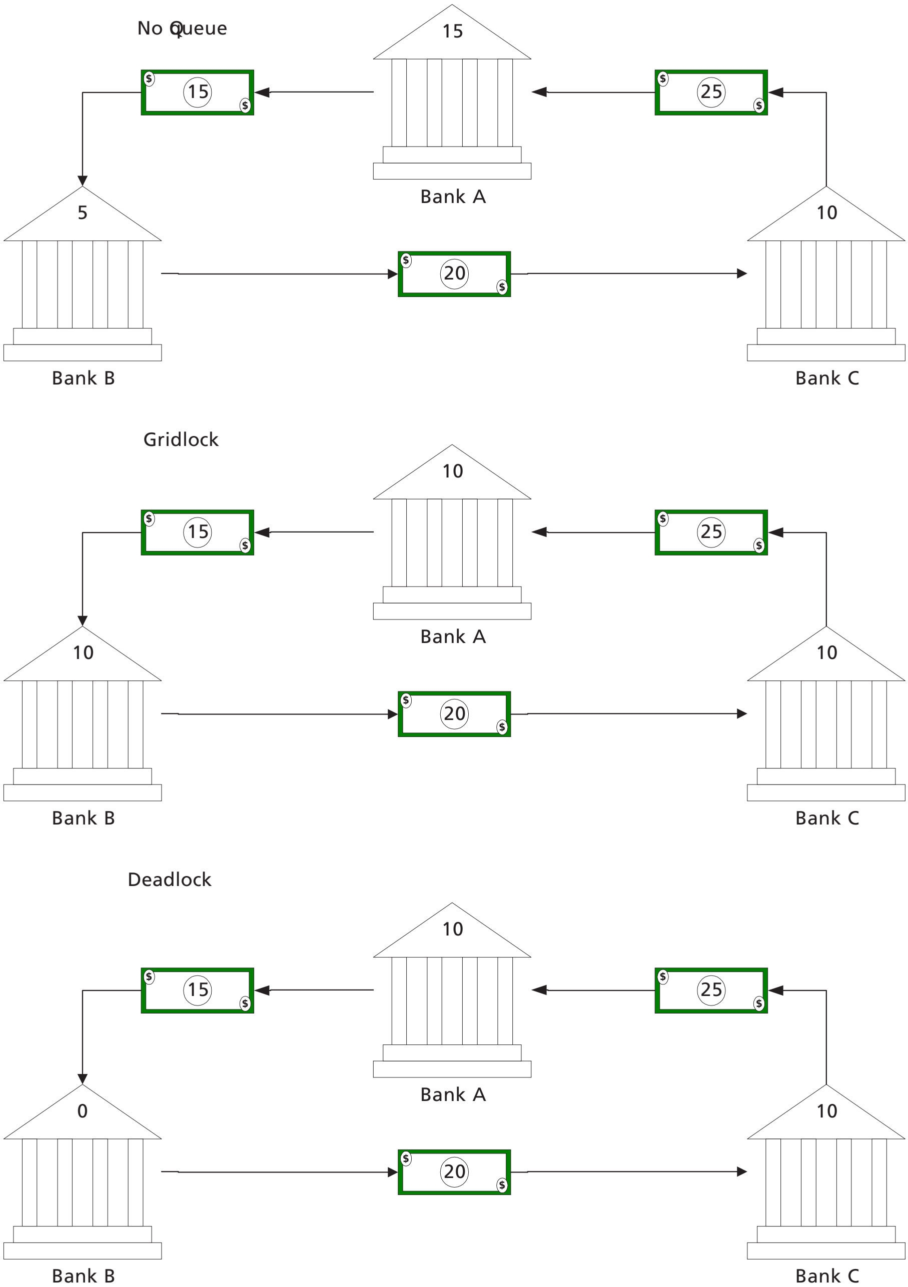
In the second instance, bank A's settlement account balance has been reduced by kr. 5, and bank B's funds have been increased equivalently. In other words, the overall liquidity in the system is the same as in the first instance. However, this exchange of liquidity between the two banks means that the payments circle now cannot be settled.

This problem arises even though all banks will have a positive balance in their settlement accounts once all payments have been effected. In other words, the system is gridlocked, as transactions are mutually awaiting each other.

A gridlock can be resolved by settling the payments simultaneously, i.e. as one block at the same time. Hereby , the banks' balances can be calculated ex ante, and overdrafts can be avoided ex post.

However, the payments are still credited and debited individually to the banks' settlement accounts. In the example, the banks will have de-

GRIDLOCK AND DEADLOCK Chart 1




posits of respectively kr. 20 (bank A), kr. 5 (bank B) and kr. 5 (bank C) if the payments are settled simultaneously.

In the third instance, bank B has no money in its settlement account, and the payments circle cannot be effected, even if the payments were settled simultaneously. Bank B would end up with a kr. 5 overdraft. This creates a system deadlock, as more liquidity is required before settlement can take place.

This example illustrates that a gridlock may be resolved by settling the payments simultaneously, or by one or more banks supplying additional liquidity to the system. A deadlock, however, can only be resolved via an additional supply of liquidity. In the example, resolution of the deadlock would as a minimum require that bank B mobilised kr. 5. In that case it would be possible to settle the payments simultaneously, whereas bank A would also have to mobilise kr. 5 if payments were to be settled individually.

## GRIDLOCK RESOLUTION

In the real world, the number of both banks and payments is naturally greater than three, and the problem of identifying and resolving gridlocks can become fairly complex. For practical purposes, gridlock resolution requires that the central bank has access to information about pending payments. For instance, banks may use a built-in queuing feature in the RTGS system.

The challenge for optimal resolution is to find the largest subset of pending payments that can be settled without any bank ending up with an overdraft (or if an overdraft facility is available as in Kronos, exceeding the overdraft limit). Furthermore, banks often wish to settle payments in a specific order, as some payments are more important than others. The central bank thus cannot pick and choose among pending payments, but must respect the priorities defined by the banks.

The Department of Informatics and Mathematical Modelling at the Technical University of Denmark has assisted Danmarks Nationalbank in developing an algorithm to resolve this problem. On the basis of the payments queued in KRONOS, the algorithm selects the largest subset which can be settled simultaneously without any bank incurring an overdraft and without deviating from the banks' requested settlement order.

The algorithm always finds the optimum solution and is fair in that the solution does not favour any bank(s). Moreover, the algorithm is so fast that settlement of payments is not delayed as a result of Danmarks Nationalbank's attempts to resolve the gridlock. The algorithm has been described by Bech and Soramäki (2001).

## SIMULATIONS

The purpose of simulations is firstly to illustrate the relationship between the liquidity available within the system and the delay of payments, and secondly to illustrate the effect of implementing the above gridlock resolution mechanism.

Simulations were conducted by running 3-4 months of actual payments data from the Danish and Finnish systems through a computer program that simulates the handling and bookkeeping of payments in an RTGS system[1]. The simulations comprise a number of different scenarios for each country, with varying liquidity available to the participants and with the mechanism activated or deactivated.

The liquidity available is measured relative to an upper and a lower bound. The lower bound (LB) corresponds to the liquidity required by the system if all payments are to be settled collectively at the close of the day. In other words, the lower bound corresponds to the liquidity requirement in a netting system with end-of-day settlement. The upper bound (UB), on the other hand, is the amount of liquidity required to settle all payments immediately. Six liquidity levels were operated with for simulation purposes. They were calculated as follows:

$$L(\alpha) = UB - \alpha(UB - LB) \qquad (1)$$

where $\alpha$ = { 0 , 0.2 , 0.4 , 0.6 , 0.8 , 1}. A liquidity level below the lower bound implies that some payments cannot be settled, and a liquidity level above the upper bound implies that the additional liquidity is never used. In the simulations below the liquidity available has been calculated as a percentage of the total value of transactions in the course of the day.

The delay in the settlement of payments was calculated using an indicator, $\rho$[2]. If all payments are settled immediately, $\rho = 0$, whereas $\rho = 1$ if all payments wait until the close of the day. The expected trade-off between liquidity and delay is illustrated in Chart 2. It is easiest to read the chart by starting with a liquidity level equivalent to the upper bound and then looking at the consequences of reducing the liquidity by approaching the lower bound.

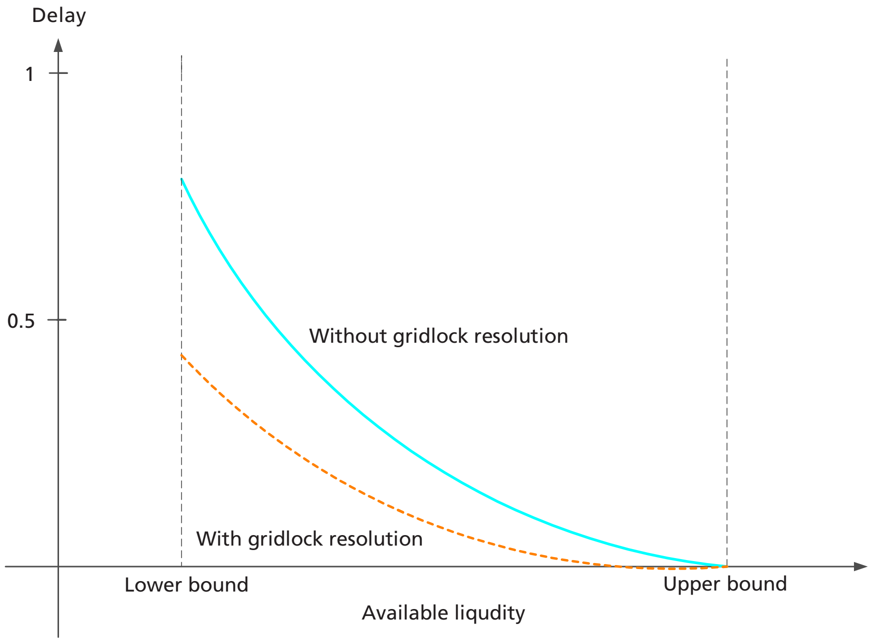
The marginal increase in the delay is seen to rise as liquidity is reduced, and *ceteris paribus* the indicator for delay (y-axis) is lower in systems with a gridlock resolution mechanism. In addition, the effect of gridlock resolution is expected to be greater when liquidity is scarce.

## DATA

Simulations were made with data from both the DN Inquiry and Transfer System and the Finnish BoF[3]-RTGS system. The DN Inquiry and Transfer

1 For a description of the simulator used, see Koponen and Soramäki (1999).
2 See Bech and Soramäki (2001) for a detailed description of the delay indicator ρ.
3 Bank of Finland.

EXPECTED TRADE-OFF BETWEEN LIQUIDITY AND DELAY Chart 2




System is the former RTGS system for Danish kroner, while BoF-RTGS is part of TARGET and therefore operates in euro.

**DN Inquiry and Transfer System**

The Danish data set comprises account entries from the last three months of 1999, equivalent to 64 business days. The data set purely comprised transfers between settlement accounts, and not e.g. transfers to the special settlement accounts used in connection with the daily settlements in the Danish Securities Centre (VP) and the retail clearing[1]. In the period analysed, 146 settlement-account holders sent or received payments. Daily turnover fluctuated between kr. 10 billion and kr. 103 billion, with a daily average of kr. 63 billion. The low turnover on some days could be attributed to Christmas and New Year. The number of payments settled per day fluctuated between 490 and 2,342, with an average of 925. There was a high degree of concentration in that the three largest participants accounted for almost 90 per cent of the total value of the payments.

[1] See Financial Institutions' Accounts at and Pledging of Collateral to Danmarks Nationalbank, page 23 of this *Monetary Review.*

TURNOVER IN DN INQUIRY AND TRANSFER SYSTEM AND BoF-RTGS — Table 1

| Billion euro | DN Min. | DN Max. | DN Avg. | BoF-RTGS Min. | BoF-RTGS Max. | BoF-RTGS Avg. |
|---|---|---|---|---|---|---|
| Individual transaction........... | 0.001 | 1,227 | 10 | 0.001 | 2,098 | 10 |
| Daily turnover....................... | 1,358 | 13,783 | 9,352 | 4,638 | 32,718 | 15,045 |
| Daily no. of transactions ..... | 490 | 2,342 | 925 | 558 | 1,872 | 1,428 |

Source: Own calculations.

**BoF-RTGS**

The Finnish data set comprises account entries from the last 100 banking days of 2000. The Finnish system had 13 participants in the period analysed, and the number of account holders was thus considerably lower than in Denmark. In Finland, the respective associations of savings banks and co-operative banks act as central clearing institutes for their members. This reduces the number of direct members of the Finnish system.

Daily turnover in the Finnish system fluctuated between euro 4.6 billion and euro 32.7 billion, with an average of euro 15 billion. Approximately 32 per cent of the turnover was related to cross-border TARGET payments.

The number of payments per day fluctuated between 558 and 1,872, with an average of 1,428. These figures did not include transfers in connection with the Finnish equivalent of the retail clearing, and cross-border TARGET payments to and from the Bank of Finland. The average turnover in the Finnish system was thus somewhat greater than in the Danish system, cf. Table 1. Average payments were in the same size range in the two systems, i.e. approximately euro 10 million or kr. 74 million.

## RESULTS

When interpreting the results of the simulations it is important to bear in mind that data reflects the banks' choices as to e.g. timing of payments on a given day, taking into account the liquidity available. It is therefore highly probable that these choices would be different under other circumstances.

On average, the lower bound totalled respectively 10.7 per cent and 4.3 per cent of the total value of transactions for the Danish and Finnish systems, cf. Table 2. The corresponding upper bounds were 37.2 per cent and 27.4 per cent. For both countries, the span between the upper and lower bounds was in the range of 25 percentage points, which illustrates that an RTGS system requires considerably more liquidity than a netting

UPPER AND LOWER LIQUIDITY BOUNDS — Table 2

| Billion euro | DN | | | BoF-RTGS | | |
|---|---|---|---|---|---|---|
| | Min. | Max. | Avg. | Min. | Max. | Avg. |
| Upper liquidity bound .................. | 634 | 4,925 | 3,421 | 639 | 5,957 | 2,746 |
| - in per cent of payment flow .... | 29.2 | 50.7 | 37.2 | 15.9 | 48.9 | 27.4 |
| Lower liquidity bound .................. | 269 | 2,276 | 958 | 11 | 3,233 | 423 |
| - in per cent of payment flow .... | 4.1 | 24.0 | 10.7 | 0.1 | 26.6 | 4.3 |

Note: The lower bound equals the liquidity requirement in a netting system with end-of-day settlement. The upper bound equals the liquidity requirement in an RTGS system with immediate settlement of all payments.
Source: The table is based on data from the 4th quarter of 1999 for the DN Inquiry and Transfer System, and for BoF-RTGS the period from September up to and including December 2000.

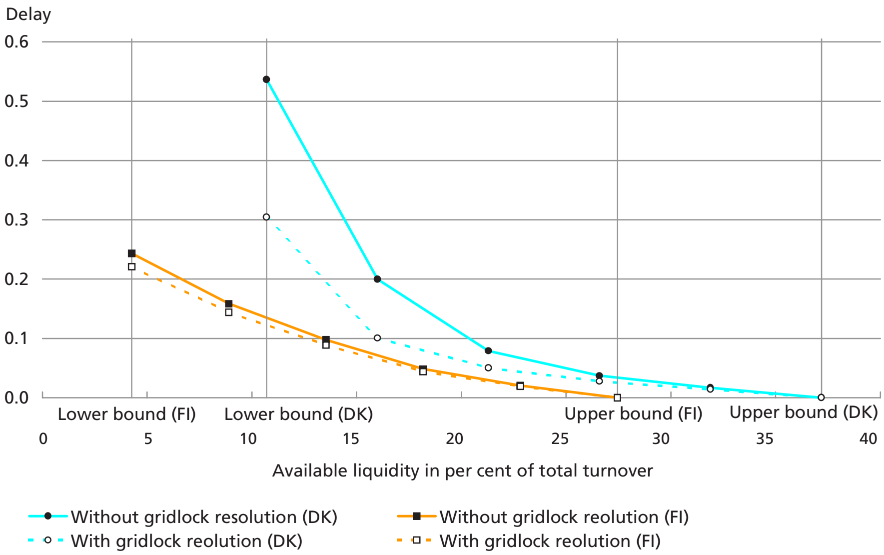
system to be able to settle the same volume of payments. On the other hand the payments are settled much sooner.

The lower values for both the lower and upper bounds for the Finnish system are, among other things, attributable to the smaller number of participants. This means that liquidity is spread among fewer participants and is thereby, all other things being equal, re-used to a higher degree during the day.

**Trade-off between liquidity and delay**

The average trade-off between liquidity and delay for the two possible system configurations is shown in Chart 3. The horizontal axis shows the liquidity available within the system relative to the total value of transactions on the day in question. The vertical axis shows the delay indicator, ρ, described above.

TRADE-OFF BETWEEN LIQUIDITY AND DELAY — Chart 3



All simulated curves are convex, reflecting the expected decrease in the marginal effect of increased liquidity in terms of delayed settlement of payments. As Chart 3 also illustrates, the proposed gridlock resolution mechanism reduces this delay in payments settlement at all liquidity levels. The greatest reduction is achieved in a situation with very scarce liquidity. Moreover, the mechanism is seen to be considerably more effective in the Danish system than in the Finnish system.

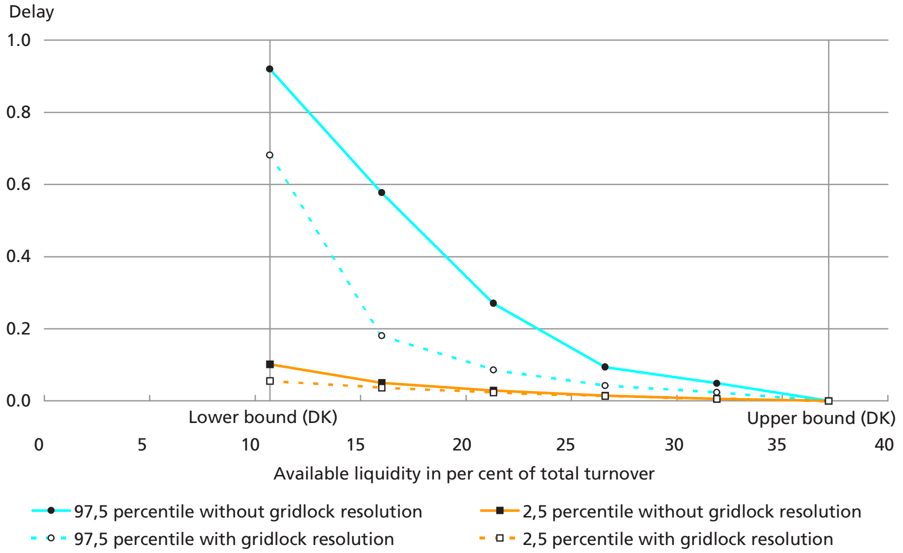
One reason for this is that the Finnish system is part of TARGET. This means that a number of the payments sent by Finnish banks via BoF-RTGS are to banks in other European countries. As payments arriving from the TARGET network do not queue in BoF-RTGS, these payments were never included in the gridlock resolution as they were settled immediately. This means that from a liquidity point of view, out-going cross-border payments are a dead weight in terms of gridlock resolution.

Not only does the delay increase when system liquidity is reduced, the variation in the delay from day to day also increases. This is illustrated for the Danish system in Chart 4, where the variation in the delay is measured as the span between the 2.5 and 97.5 percentiles for daily delays.

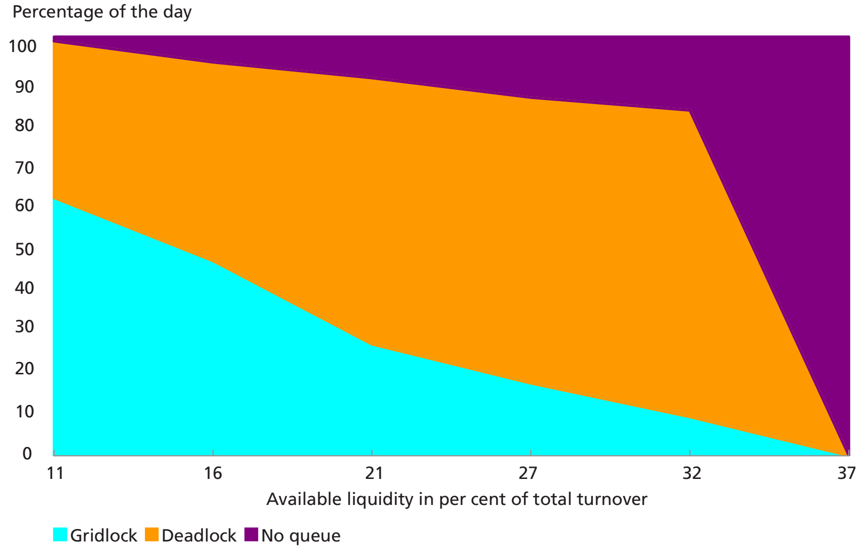
The span increases as liquidity is reduced. This reflects the fact that scarce liquidity affects settlement differently from day to day. On some days it is of no importance, as the 2.5 percentile only increases marginally when liquidity is reduced. On other days, however, considerable delays occur, which is reflected in a significant increase in the 97.5 per-

VARIATION IN DELAY

Chart 4



NO QUEUE, GRIDLOCK AND DEADLOCK (DANISH DATA) Chart 5




centile when liquidity becomes scarce. The gridlock resolution mechanism is found to considerably reduce the number of "problematic" days, i.e. days when settlement is delayed considerably.

In addition to quantifying the trade-off between liquidity and delay, the simulations also describe the state of the system, cf. Chart 5. The Chart shows the percentage of the day when the system respectively has no queues, is gridlocked or is deadlocked. At the upper bound, by definition there is no queue at any time of the day. As liquidity is reduced, first primarily deadlocks begin to appear as only few transactions are queued. Later on, the number of gridlocks increases as more transactions are queued and some of them mutually await each other's settlement.

At the lower bound, the Danish system is gridlocked for more than 60 per cent of the day. In a system with a gridlock resolution mechanism, the system either has no queues, or is in a deadlock, which can only be resolved by supplying additional liquidity.

## SUMMARY

In an RTGS system, situations may arise where payments mutually await each other's settlement. Such delays in the settlement of payments can be reduced by using a suitable algorithm. Simulation runs using Finnish and Danish data showed that the algorithm implemented in KRONOS can reduce such delays, especially on days with scarce liquidity.

At present, gridlocks are not deemed to be a major problem in the Danish system. This view is supported by the fact that close to 80 per cent of DN Inquiry and Transfer System payments were settled by noon on a normal day[1]. The reason is that the banks have considerable bond holdings, which can be used as collateral, so that the alternative cost of raising liquidity is assessed to be relatively low on a normal day.

However, the situation may be changing. The trend is for the banks to have to provide collateral in a growing number of cases and that they must be able to raise liquidity at shorter notice. Consequently, it is not improbable that we may see 'crisis' days or periods of the day where liquidity is scarce. In this type of situation, the simulations show that the gridlock resolution mechanism is effective in preventing delays in the settlement of payments and in ensuring smoother operation of the payment system.

[1] See Financial Stability, Danmarks Nationalbank, *Monetary Review*, 2nd Quarter 2001, page 77.

## APPENDIX

In order to provide a formal definition of gridlock, deadlock and the gridlock resolution problem, a little mathematical notation is necessary. Assume that there are *n* banks, and let $Q_i$ be the set of payments in bank *i*'s queue. The total queue in the RTGS system is expressed as $Q = \cup_{i=1}^{n} Q_i$. In the same way, the subset of payments to be settled simultaneously is expressed as $X = \cup_{i=1}^{n} X_i$, where $X_i$ is the contribution from the individual bank's queue. The ex ante and ex post balances including overdraft facilities on the individual bank's settlement account are expressed as $B_i$ and $B_i(\circ)$, respectively. The value of the payments received by bank *i* is expressed as $R(X_{-i})$, and the value of the payments remitted by bank *i* is expressed as $S(X_i)$. Let $\succ_i$ express the preference relation for bank *i* in terms of the order in which payments are to be settled.

**Definition 1 (Gridlock)**
*A gridlock is a situation where $Q \neq \emptyset$ and there is a non-empty subset $X \subseteq Q$, which means that if the payments in $X$ are settled simultaneously, then*

$$B_i(B_i, X) = B_i - S(X_i) + R(X_{-i}) \geq 0, \quad for\ i\ =\ 1,\ldots,n \qquad (2)$$

and

$$\forall\, x \in X_i\ \nexists\, q \in Q_i \setminus X_i \text{ so that } q \succ_i x,\ for\ i\ =\ 1,\ldots,n \qquad (3)$$

The first condition (the liquidity condition) stipulates that if the payments in X are settled simultaneously, the ex post balance would not be negative for any bank. The ex post balance is expressed as the ex ante balance less the value of payments remitted by bank *i* $S(X_i)$ plus the value of payments received by bank *i* $R(X_{-i})$. The second condition stipulates that the priority by which banks want payments to be settled must be observed.

**Definition 2 (Deadlock)**
*A deadlock is a situation where $Q \neq \emptyset$, and X, as defined in definition 1, is empty, i.e. $X = \emptyset$*

The gridlock resolution problem consists of selecting the largest possible subset of payments queued that can be settled simultaneously without breaking any of the two conditions in equations (2) and (3).

**Definition 3 (Gridlock resolution)**
*Let V(X) express the value or number of transactions in X. The gridlock resolution is* $\max_{X \subseteq Q} V(X)$*, provided that the liquidity condition stated in (2) and the priority condition stated in (3) are observed.*

The solution to the problem in definition 3 is the same whether the value or the number of transactions is applied.

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